## VECTORS

1.1-1.4: Introduction to the basic concepts of Vectors

Task 1 : Write down all the vectors as shown in each of the following diagrams.


Task 2 : Determine the vectors that are equal in each of the following diagrams.


Task 3 : Determine the negative vectors as shown in the following diagrams.
(1)



The negative vector of $\underset{\sim}{p}$ :

$$
-\underset{\sim}{p}=
$$

$\overrightarrow{V W}=$


The negative vector of $\underset{\sim}{c}$ :
$-\underset{\sim}{c}=$

## Answers:

Task 1 :
(1) $\overrightarrow{A B}=\underset{\sim}{x} ; \underset{\sim}{y}=\overrightarrow{P Q}$
(2) $\overrightarrow{G H}=\underset{\sim}{b} ; \underset{\sim}{a}=\overrightarrow{E F}$
(3) $\overrightarrow{A B}=\underset{\sim}{a} ; \underset{\sim}{x}=\overrightarrow{P Q} ; \overrightarrow{E F}=\underset{\sim}{c} ; \underset{\sim}{b}=\overrightarrow{I J}$

Task 2 :
(1) $\underset{\sim}{a}=\underset{\sim}{q}$
(2) $\underset{\sim}{a}=f$
(3) $\underset{\sim}{a}=\underset{\sim}{d} ; \underset{\sim}{b}=\underset{\sim}{e} ; \underset{\sim}{c}=\underset{\sim}{f} ; \overrightarrow{A B}=\overrightarrow{G H} ; \overrightarrow{C D}=\overrightarrow{M N} ; \overrightarrow{E F}=\overrightarrow{K L}$

Task 3 :
(1) $\overrightarrow{X Y}=-\underset{\sim}{x} ; \overrightarrow{S T}=-\underset{\sim}{a} ; \overrightarrow{C D}=-\underset{\sim}{c} ; \overrightarrow{V W}=-\underset{\sim}{b}$
(2) $-\underset{\sim}{p}=\underset{\sim}{q}$
(3) $-\underset{\sim}{c}=\underset{\sim}{a}$

## VECTORS

1.5 Multiplication of Vectors by Scalars

Task 1 : State the following vectors in terms of $a$.
(1)

(a) $\overrightarrow{A B}=$
(b) $\overrightarrow{C D}=$
(3)

(a) $\overrightarrow{P Q}=$
(b) $\overrightarrow{R S}=$
(5)

(a) $\overrightarrow{A B}=$
(b) $\overrightarrow{C D}=$

(a) $\overrightarrow{E F}=$
(4)

(a) $\overrightarrow{E F}=$
(b) $\overrightarrow{G H}=$
(6)

(a) $\overrightarrow{P Q}=$
(b) $\overrightarrow{R S}=$

Answers: (1)(a) $\overrightarrow{A B}=2 \underset{\sim}{a}$ (b) $\overrightarrow{C D}=3 \underset{\sim}{a}$ (2)(a) $\overrightarrow{E F}=2 \underset{\sim}{b}$ (b) $\overrightarrow{G H}=\frac{3}{2} \underset{\sim}{b} \quad$ (3) (a) $\overrightarrow{P Q}=3 \underset{\sim}{c}$ (b) $\overrightarrow{R S}=2 \underset{\sim}{c}$
(4) (a) $\overrightarrow{E F}=2 \underset{\sim}{d}$ (b) $\overrightarrow{G H}=3 \underset{\sim}{d}$
(5) (a) $\overrightarrow{A B}=2 \underset{\sim}{e}$ (b) $\overrightarrow{C D}=\frac{3}{2} \underset{\sim}{e}$
(6) (a) $\overrightarrow{P Q}=-2 \underset{\sim}{f}$ (b) $\overrightarrow{R S}=-\frac{1}{2} \underset{\sim}{f}$

Task 2 : In each diagram below, determine the vectors that are parallel and state their relationships.


## VECTORS

2.1-2.4 Addition and Subtraction of Vectors.

Task 1 : Determine the resultant vector of two or more parallel vectors by addition and subtraction operations.

| (1) $2 \underset{\sim}{a}+\underset{\sim}{a}+\frac{1}{2} \underset{\sim}{a}$ | (2) $\frac{1}{2} \underset{\sim}{x}+2 \underset{\sim}{x}+\frac{1}{3} \underset{\sim}{x}$ | (3) $5 \underset{\sim}{y}+\frac{1}{2} \underset{\sim}{y}+\frac{3}{4} \underset{\sim}{y}$ |
| :---: | :---: | :---: |
|  |  |  |
| (4) $5 \underset{\sim}{b}-3 \underset{\sim}{b}$ | (5) $7 \underset{\sim}{a}-3 \underset{\sim}{a}-\frac{1}{2} \underset{\sim}{a}$ | (6) $12 \underset{\sim}{b}-2 \underset{\sim}{b}-5 \underset{\sim}{b}$ |
| $2 \sim_{\sim}$ | $\frac{7}{2} a$ | $5{ }_{\sim}$ |
| (7) $2 \underset{\sim}{a}+\frac{1}{3} \underset{\sim}{b}+\frac{1}{2} \underset{\sim}{a}+\underset{\sim}{b}$ | (8) $\underset{\sim}{a}+2 \underset{\sim}{b}+3 \underset{\sim}{a}+\frac{1}{4} \underset{\sim}{b}$ | (9) $4 \underset{\sim}{u}+\frac{1}{2} \underset{\sim}{v}+\frac{1}{5} \underset{\sim}{u}+\frac{1}{6} \underset{\sim}{v}$ |
| $\frac{5}{2} \underset{\sim}{a}+\frac{4}{3} \underset{\sim}{ }$ | $4 \underset{\sim}{a}+\frac{9}{4} \underset{\sim}{b}$ | $\frac{21}{5} \underset{\sim}{u}+\frac{2}{3} \underset{\sim}{v}$ |
| (10) $6 \underset{\sim}{x}-4 \underset{\sim}{y}-\underset{\sim}{x}+2 \underset{\sim}{y}$ | (11) $4 \underset{\sim}{u}+5 \underset{\sim}{v}-2 \underset{\sim}{u}-3 \underset{\sim}{v}$ | (12) $6 \underset{\sim}{s}-8 \underset{\sim}{t}-9 \underset{\sim}{s}-2 \underset{\sim}{t}$ |
| $5 x-2 y$ | $2 \underset{\sim}{u}+2 \underset{\sim}{v}$ | $-3 \underset{\sim}{s}-10 \underset{\sim}{t}$ |

Task 2 : Determine the resultant vector of two or more non-parallel vectors by addition and subtraction operations.
(1)

(a) $\underset{\sim}{a}+\underset{\sim}{b}=$
(b) $\underset{\sim}{b}+\underset{\sim}{c}=$
(c) $\underset{\sim}{c}+\underset{\sim}{d}=$
(d) $\overrightarrow{A B}+\overrightarrow{B D}=$
(2)

(a) $\underset{\sim}{x}+\underset{\sim}{y}=$
(b) $\underset{\sim}{w}+\underset{\sim}{x}=$
(c) $\overrightarrow{D C}+\overrightarrow{C B}=$
(d) $\overrightarrow{B C}+\overrightarrow{C A}=$
(3)

(a) $\underset{\sim}{a}+\underset{\sim}{b}=$
(b) $\underset{\sim}{b}+\underset{\sim}{c}=$
(c) $\overrightarrow{Q P}+\overrightarrow{P T}=$
(d) $\overrightarrow{R T}+\overrightarrow{T Q}=$
(4) ABCD in the diagram is a parallelogram.

(a) $\overrightarrow{A B}+\overrightarrow{A D}=$
(b) $\overrightarrow{B A}+\overrightarrow{B C}=$
(c) $\overrightarrow{D A}+\overrightarrow{D C}=$
(d) $\overrightarrow{C B}+\overrightarrow{C D}=$
(5) PQRS in the diagram is a parallelogram.

(a) $\overrightarrow{P Q}+\overrightarrow{P S}=$
(b) $\overrightarrow{S P}+\overrightarrow{S R}=$
(c) $\overrightarrow{Q P}+\overrightarrow{Q R}=$
(d) $\overrightarrow{R Q}+\overrightarrow{R S}=$
(6) ABCDEF in the diagram is a regular Hexagon
(a) $\overrightarrow{A B}+\overrightarrow{A F}=$

(b) $\overrightarrow{O C}+\overrightarrow{O E}=$
(c) $\overrightarrow{F A}+\overrightarrow{F E}=$
(d) $\overrightarrow{C B}+\overrightarrow{C D}=$

| (7) ABCDE in the diagram is a pentagon. <br> (a) $\overrightarrow{A B}+\overrightarrow{B C}+\overrightarrow{C D}$ <br> (b) $\overrightarrow{A C}+\overrightarrow{C D}+\overrightarrow{D E}$ $=$ <br> (c) $\overrightarrow{C D}+\overrightarrow{D A}+\overrightarrow{A E}$ $=$ | (8) PQRST in the diagram is a pentagon. <br> (a) $\overrightarrow{P Q}+\overrightarrow{Q R}+\overrightarrow{R T}$ <br> (b) $\overrightarrow{R S}+\overrightarrow{S T}+\overrightarrow{T Q}$ $=$ <br> (c) $\overrightarrow{T P}+\overrightarrow{P R}+\overrightarrow{R Q}$ |
| :---: | :---: |
| (9) KLMNOP in the diagram is a hexagon. <br> (a) $\overrightarrow{P O}+\overrightarrow{O N}+\overrightarrow{N M}+\overrightarrow{M L}$ <br> (b) $\overrightarrow{K M}+\overrightarrow{M N}+\overrightarrow{N P}+\overrightarrow{P L}$ <br> $=$ <br> (c) $\overrightarrow{M N}+\overrightarrow{N L}+\overrightarrow{L P}+\overrightarrow{P N}$ | (10) ABCDE in the diagram is a pentagon. $\text { (a) } \begin{aligned} & \overrightarrow{A D}-\overrightarrow{C D} \\ &= \\ &= \\ & \text { (b) } \overrightarrow{C B}-\overrightarrow{E B} \\ &= \\ &= \end{aligned}$ <br> (a) |
| (11) PQRST in the diagram is a pentagon. <br> (a) $\overrightarrow{P S}-\overrightarrow{T S}$ <br> (b) $\overrightarrow{R Q}-\overrightarrow{P Q}$ <br> (c) $\overrightarrow{S P}-\overrightarrow{T P}$ | (12) ABCDEF in the diagram is a hexagon. <br> (a) $\overrightarrow{A D}-\overrightarrow{D C}$ $=$ = <br> (b) $\overrightarrow{B C}-\overrightarrow{D C}-\overrightarrow{E D}=$ <br> (c) $\overrightarrow{C A}-\overrightarrow{F A}-\overrightarrow{E F}=$ |
| (13) ABCDEF in the diagram is a regular hexago <br> (a) $\underset{\sim}{a}-\underset{\sim}{b}=$ <br> (b) $\underset{\sim}{b}-2 \underset{\sim}{a}=$ <br> (c) $\underset{\sim}{b}-\underset{\sim}{c}=$ | with centre O. <br> (d) $\underset{\sim}{a}-2 \underset{\sim}{b}=$ <br> (e) $\underset{\sim}{a}-\underset{\sim}{c}=$ <br> (f) $\underset{\sim}{c}-2 \underset{\sim}{b}=$ |

## VECTORS

2.5 Represent Vectors as Linear Combination of other Vectors.

Task 1 : Express the following vectors in terms of $\underset{\sim}{x}$ and $\underset{\sim}{y}$.

Answers:
(1) $\overrightarrow{A C}=\underset{\sim}{x}+\underset{\sim}{y}$
(2) $\overrightarrow{Q R}=-\underset{\sim}{y}+\underset{\sim}{x}$
(3) $\overrightarrow{T Q}=-2 \underset{\sim}{x}+\underset{\sim}{y} ; \overrightarrow{P R}=\underset{\sim}{x}+\frac{1}{2} \underset{\sim}{y}$
(4) $\overrightarrow{E G}=2 \underset{\sim}{x}+\underset{\sim}{y}$
(5) $\overrightarrow{F G}=-\underset{\sim}{x}+2 \underset{\sim}{y}$
(6) $\overrightarrow{B C}=-\underset{\sim}{y}+\underset{\sim}{x} ; \overrightarrow{A D}=\frac{1}{2} \underset{\sim}{y}+\frac{1}{2} \underset{\sim}{x}$

Task 2 : For each of the following diagrams, express the vector $\overrightarrow{P Q}$ in terms of $\underset{\sim}{x}$ and $\underset{\sim}{y}$.



## VECTORS

3.1 Express Vectors in Cartesian Plane in the form of $x \underset{\sim}{i}+y \underset{\sim}{j}$ or $\binom{x}{y}$.

Task : Express the following vectors in the form of $x \underset{\sim}{i}+y \underset{\sim}{j \operatorname{jor}}\binom{x}{y}$.

(4) $(-6,4) ;\binom{-6}{4} ;-6 \underset{\sim}{i}+4 \underset{\sim}{j}$
(5) $(-5,3) ;\binom{-5}{3} ;-5 \underset{\sim}{i}+3 \underset{\sim}{j}$
(6) $(-4,4) ;\binom{-4}{4} ;-4 \underset{\sim}{i}+4 \underset{\sim}{j}$
(7) $(5,-4) ;\binom{5}{-4} ; 5 \underset{\sim}{i}-4 \underset{\sim}{j}$
(8) $(7,-3) ;\binom{7}{-3} ; 7 \underset{\sim}{i}-3 \underset{\sim}{j}$

Vectors


Answers: $(9)(6,-3) ;\binom{6}{-3} ; \underset{\sim}{i}-3 \underset{\sim}{j}(10)(-5,-6) ;\binom{-5}{-6} ;-5 \underset{\sim}{i}-6 \underset{\sim}{j}(11)(-7,-4) ;\binom{-7}{-4}$;
$-7 \underset{\sim}{i}-4 \underset{\sim}{j}(12)(-6,-3) ;\binom{-6}{-3} ;-6 \underset{\sim}{i}-3 \underset{\sim}{j}(13)\binom{4}{1}=4 \underset{\sim}{i}+\underset{\sim}{j} ;\binom{-2}{3}=-2 \underset{\sim}{i}+3 \underset{\sim}{j}(14)\binom{6}{2}=$
$6 \underset{\sim}{i}+2 \underset{\sim}{j} ;\binom{3}{-4}=\underset{\sim}{i}-4 \underset{\sim}{j}(15)\binom{1}{4}=\underset{\sim}{i}+4 \underset{\sim}{j} ;\binom{-5}{2}=-5 \underset{\sim}{i}+2 \underset{\sim}{j}(16)\binom{4}{-8}=4 \underset{\sim}{i}-8 \underset{\sim}{j} ;\binom{-3}{-6}=$
$-3 \underset{\sim}{i}-6 \underset{\sim}{j}$

## VECTORS

3.2 - 3.3Determine the Unit Vectors in the Direction of given Vectors.

Task 1 : For each of the following vectors in terms of $\underset{\sim}{i}$ and $\underset{\sim}{j}$, find the magnitude and the unit vector in the direction of the given vector.


Task 2 : For each of the following vectors in the form $\binom{x}{y}$, find the magnitude and the unit vector in the direction of the given vector.

| (1) $\overrightarrow{P Q}=\binom{8}{6}$ | (2) $\overrightarrow{S T}=\binom{15}{-8}$ | (3) ) $\overrightarrow{C D}=\binom{-6}{8}$ |
| :---: | :---: | :---: |
| magnitude of $\overrightarrow{P Q}$ | magnitude of $\overrightarrow{S T}$ | magnitude of $\overrightarrow{C D}$ |
| $\|\overrightarrow{P Q}\|=$ | $\|\overrightarrow{S T}\|=$ | $\|\overrightarrow{C D}\|=$ |
| Unit vector in the direction of $\overrightarrow{P Q}$ $\frac{\wedge}{P Q}=$ | Unit vector in the direction of $\overrightarrow{S T}$ $\hat{\overrightarrow{S T}}=$ | Unit vector in the direction of $\overrightarrow{C D}$ $\frac{\hat{C D}}{}=$ |
| $10 ;\binom{\frac{4}{5}}{\frac{3}{5}}$ | 17 ; $\binom{\frac{15}{17}}{-\frac{8}{17}}$ | $10 ;\binom{-\frac{3}{5}}{\frac{4}{5}}$ |
| (4) $\overrightarrow{V W}=\binom{-9}{-40}$ | (5) $\underset{\sim}{u}=\binom{-12}{-9}$ | (6) $\underset{\sim}{v}=\binom{-24}{7}$ |
| magnitude of $\overrightarrow{V W}$ | magnitude of $\underset{\sim}{u}$ | magnitude of $\underset{\sim}{v}$ |
| $\|\overrightarrow{V W}\|=$ | $\|\underset{\sim}{u}\|=$ | $\|\underset{\sim}{v}\|=$ |
| Unit vector in the direction of $\overrightarrow{V W}$ $\stackrel{\wedge}{V W}=$ | Unit vector in the direction of $\underset{\sim}{u}$ $\underset{\sim}{\hat{u}}=$ | Unit vector in the direction of $\underset{\sim}{v}$ $\underset{\sim}{\hat{v}}=$ |
| $41 ;\binom{-\frac{9}{41}}{-\frac{40}{41}}$ | $15 ;\binom{-\frac{4}{5}}{-\frac{3}{5}}$ | $25 ;\binom{-\frac{24}{25}}{\frac{7}{25}}$ |

## VECTORS

3.4-3.7 Addition, Subtraction and Multiplication of Vectors

Task 1 : Given $\underset{\sim}{a}=2 \underset{\sim}{i}+5 \underset{\sim}{j}, \underset{\sim}{b}=\underset{\sim}{i}-4 \underset{\sim}{j}$ and $\underset{\sim}{c}=-3 \underset{\sim}{i}+7 \underset{\sim}{j}$, find the following vectors in terms of $\underset{\sim}{i}$ and $\underset{\sim}{j}$.

| $\text { (1) } \begin{gathered} 2 \underset{\sim}{b} \\ = \end{gathered}$ | $2 \underset{\sim}{i}-8 \underset{\sim}{j}$ | (2) $\begin{aligned} & 3 \underset{\sim}{3 a} \\ & = \end{aligned}$ | $6 \underset{\sim}{i}+15 \underset{\sim}{j}$ | (3) $\begin{aligned} & { }_{\sim}^{\underset{\sim}{c}} \\ & = \end{aligned}$ | $-12 \underset{\sim}{i}+28 \underset{\sim}{j}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\text { (4) } \begin{gathered} \frac{1}{2} \underset{\sim}{a} \\ = \end{gathered}$ | $\underset{\sim}{i}+\frac{5}{2} \underset{\sim}{j}$ | $\begin{gathered} \text { (5) } \underset{\sim}{a}+3 \underset{\sim}{b} \\ = \end{gathered}$ | $5 \underset{\sim}{i}-7 \underset{\sim}{j}$ | $\begin{gathered} \text { (6) } 2 \underset{\sim}{a}+\underset{\sim}{b} \\ = \end{gathered}$ | $5 \underset{\sim}{i}+6 \underset{\sim}{j}$ |
| (7) $\underset{\sim}{b}+\underset{\sim}{c}$ <br> $=$ | $-2 \underset{\sim}{i}-3 \underset{\sim}{j}$ | (8) $\begin{aligned} & \frac{1}{2} \underset{\sim}{b}+\underset{\sim}{a} \\ & = \end{aligned}$ | $\frac{5}{2} \underset{\sim}{i}+3 \underset{\sim}{j}$ | $\text { (9) } \begin{gathered} \underset{\sim}{a}-3 \underset{\sim}{b} \\ = \end{gathered}$ | $-\underset{\sim}{i}+17 \underset{\sim}{j}$ |
| $\text { (10) } 4 \underset{\sim}{b}-\underset{\sim}{c}$ $=$ | $7 \underset{\sim}{i}-23 \underset{\sim}{j}$ | (11) $\begin{aligned} & 3 \underset{\sim}{c}-2 \underset{\sim}{a} \\ & = \end{aligned}$ | $-13 \underset{\sim}{i}+11 \underset{\sim}{j}$ | (12) $\begin{aligned} & \underset{\sim}{b}-\frac{1}{2} \underset{\sim}{a} \\ & = \end{aligned}$ | $-\frac{13}{2} \underset{\sim}{j}$ |

Task 2 : Given $\underset{\sim}{a}=\binom{3}{4}, \underset{\sim}{b}=\binom{-2}{5}$ and $\underset{\sim}{c}=\binom{6}{-1}$, express the following in the form of $\binom{x}{y}$.

| (1) $4 \underset{\sim}{b}$ $=$ | $\binom{-8}{20}$ | (2) $2 c$ $=$ | $\binom{12}{-2}$ | (3) $\frac{1}{2} \underset{\sim}{a}$ <br> $=$ | $\binom{\frac{3}{2}}{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| (4) $\begin{aligned} & 2 \underset{\sim}{a}+\underset{\sim}{b} \\ & = \end{aligned}$ | $\binom{4}{13}$ | $\text { (5) } \begin{gathered} \underset{\sim}{b}+3 \underset{\sim}{c} \\ = \end{gathered}$ | $\binom{16}{2}$ | $\text { (6) } \begin{gathered} 2 \underset{\sim}{c}+\underset{\sim}{a} \\ = \end{gathered}$ | $\binom{15}{2}$ |
| $\text { (7) } 3 \underset{\sim}{a}-\underset{\sim}{b}$ | $\binom{11}{7}$ | $\text { (8) } \begin{gathered} 2 b-\underset{\sim}{c} \\ = \end{gathered}$ | $\binom{-10}{11}$ | $\text { (9) } \begin{gathered} \underset{\sim}{c}-2 a \\ = \end{gathered}$ | $\binom{0}{-9}$ |
| $\begin{gathered} \text { (10) } 3 \underset{\sim}{c}-2 \underset{\sim}{b} \\ = \end{gathered}$ | $\binom{22}{-13}$ | $\begin{gathered} \text { (11) } \quad \underset{\sim}{b}+\underset{\sim}{a} \\ = \end{gathered}$ | $\binom{-3}{19}$ | $\begin{gathered} \text { (12) } 4 \underset{\sim}{a}-\underset{\sim}{c} \\ = \end{gathered}$ | $\binom{6}{17}$ |

Task 3 : Write vector $\overrightarrow{P Q}$ in the form $\binom{x}{y}$ and determine its magnitude. Hence, find the unit vector in the direction of vector $\overrightarrow{P Q}$.


Task 4 : Write vector $\overrightarrow{A B}$ in terms of $\underset{\sim}{i}$ and $\underset{\sim}{j}$ and find its magnitude. Hence, find the unit vector in the direction of vector $\overrightarrow{A B}$.

| (1) $O(0,0), A(4,-30)$ and $B(-3,-6)$. $-7 \underset{\sim}{i}+24 \underset{\sim}{j} ; 25 ;-\frac{7}{25} \underset{\sim}{i}+\frac{24}{25} \underset{\sim}{j}$ | (2) $O(0,0), A(-5,2)$ and $B(3,-4) . ~\left(\begin{array}{rl} \\ 8 \underset{\sim}{i}-6 \underset{\sim}{j} ; 10 ; \frac{4}{5} \underset{\sim}{i}-\frac{3}{5} \underset{\sim}{j}\end{array}\right.$ |
| :---: | :---: |

Task 5 : Given $\underset{\sim}{a}=3 \underset{\sim}{i}+4 \underset{\sim}{j}, \underset{\sim}{b}=2 \underset{\sim}{i}-\underset{\sim}{j}$ and $\underset{\sim}{c}=-\underset{\sim}{i}+5 \underset{\sim}{j}$, find in terms of $\underset{\sim}{i}$ and $\underset{\sim}{j}$, the unit vector in the direction of the vectors below.


